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# Some Implications of Arrow's Theorem for Voting Rights

Grant M. Hayden\*

*Arrow's theorem proves that no voting procedure can meet certain conditions of both fairness and logic. In this note, Grant Hayden explores the ramifications of the theorem for qualitative vote dilution. After describing Arrow's argument, Mr. Hayden considers four democratic voting procedures—the Condorcet method, the amendment procedure, the Borda count, and cumulative voting—in the light of the theorem. He then explores some of the theoretical and practical implications of the theorem. In the remainder of the note, Mr. Hayden discusses how well section 2 of the Voting Rights Act of 1965 and its judicial interpretation in *Thornburg v. Gingles* accord with the dictates of Arrow's theorem, ultimately concluding that the courts should consider the first two in the light of the theorem.*

## INTRODUCTION

Almost fifty years ago, Justice Frankfurter warned that legislative apportionment was a "political thicket" courts should not attempt to penetrate.<sup>1</sup> The Supreme Court, however, ignored Justice Frankfurter's admonition and plunged headlong into that thicket in *Baker v. Carr*,<sup>2</sup> a 1962 decision opening the door to challenging state voting procedures on constitutional grounds. While courts easily dispatched the problem of quantitative vote dilution with the now classic formulation "one person, one vote,"<sup>3</sup> the complexities of *qualitative* vote dilution<sup>4</sup> have proven more intractable. As a result, courts continue to struggle to develop satisfactory standards for measuring and remedying qualitative vote dilution.

As courts search for acceptable standards, their progress may be impeded by a theoretical barrier first described by Kenneth Arrow in 1951.<sup>5</sup> Arrow's

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1. *Colegrove v. Green*, 328 U.S. 549, 556 (1946), quoted in Larry Alexander, *Lost in the Political Thicket*, 41 FLA. L. REV. 563, 563 (1989).

2. 369 U.S. 186, 209 (1962) (holding that appellants' challenge to a Tennessee apportionment scheme was justiciable and presented no political question).

3. *Gray v. Sanders*, 372 U.S. 368, 381 (1963) (finding unconstitutional a Georgia law whose effect was to make each vote count less as the population of a county increased).

4. Qualitative vote dilution occurs when voters' preferences are not accurately expressed in the outcome of an election, despite the fact that society weighs each individual's vote equally.

5. KENNETH J. ARROW, *SOCIAL CHOICE AND INDIVIDUAL VALUES* 93-96 (2d ed. 1963).

theorem holds that no voting procedure can be both fair and logical.<sup>6</sup> This note examines the theorem and discusses its implications for qualitative vote dilution. Part I describes Arrow's theorem and its five conditions of fairness and logicity. Consideration of four democratic voting procedures and their violation of at least one of the theorem's conditions illustrates the power of Arrow's observation. Part II explores the theoretical and practical implications of the theorem. Finally, Part III discusses how well section 2 of the Voting Rights Act of 1965<sup>7</sup> and its judicial interpretation accord with the dictates of Arrow's theorem.

Some definitions are essential for the reader unfamiliar with social choice theory.<sup>8</sup> An *individual preference order* is a complete arrangement of a set of alternatives in order of their desirability to an individual.<sup>9</sup> The relationship between any two alternatives is either one of preference (*P*) or indifference (*I*). Thus, if Chris's preference order is  $xPyPzIw$ , then Chris prefers  $x$  to  $y$ , prefers  $y$  to  $z$ , and is indifferent between  $z$  and  $w$ .<sup>10</sup> A *preference profile* is a set of individual preference orders, one for each individual.<sup>11</sup> By contrast, a *social preference order* is a complete arrangement of alternatives in order of their attractiveness to society as a whole.<sup>12</sup> Finally, a *social choice function* translates a series of individual preference profiles into a social preference order.<sup>13</sup>

The ideal social choice function successfully aggregates individual preference orders into social preference orders, translating individual desires into group choices.<sup>14</sup> Historically, democratic institutions have adopted voting procedures to handle this task.<sup>15</sup> Unfortunately, however, the adequacy of all social choice functions was called into question with the publication of Arrow's theorem.

6. See NORMAN FROHLICH & JOE A. OPPENHEIMER, *MODERN POLITICAL ECONOMY* 19-23 (1978) (summarizing Arrow's assumptions, conditions, and conclusions). See text accompanying notes 16-32 *infra* for a detailed description of the theorem.

7. 42 U.S.C. §. 1973 (1988).

8. The terminology in this note is largely derived from the work of William Riker. See WILLIAM H. RIKER, *LIBERALISM AGAINST POPULISM: A CONFRONTATION BETWEEN THE THEORY OF DEMOCRACY AND THE THEORY OF SOCIAL CHOICE* 293-98 (1982) (providing a glossary of terms).

9. *Id.* at 296.

10. All examples in this note involve individuals who prefer one alternative to another; no individual will be indifferent.

11. RIKER, *supra* note 8, at 296.

12. See PETER C. ORDESHOOK, *GAME THEORY AND POLITICAL THEORY: AN INTRODUCTION* 55 (1986) (warning that institutions cannot be understood "as black boxes into which we plug preferences and out of which emerge . . . 'social preference'"); RIKER, *supra* note 8, at 18 (providing an example of "social preference").

13. RIKER, *supra* note 8, at 297.

14. "The theory of social choice is a theory about the way the tastes, preferences, or values of individual persons are amalgamated and summarized into the choice of a collective group or society." *Id.* at 1.

15. See FROHLICH & OPPENHEIMER, *supra* note 6, at 16-17 (using the example of public preferences over American policy in Vietnam to illustrate the difficulty of using voting procedures to find a rational group choice).

## I. ARROW'S THEOREM

Arrow's theorem demonstrates that no social choice function can simultaneously satisfy certain minimal conditions of fairness and logicity.<sup>16</sup> The theorem stipulates four fairness conditions—nondictatorship, Pareto efficiency, universal admissibility, and independence from irrelevant alternatives—and one logical condition—transitivity.<sup>17</sup> Its proscription arises whenever two or more individuals choose among three or more alternatives.<sup>18</sup>

A. *The Conditions*1. *Nondictatorship.*

The condition of nondictatorship ensures that no single person's preferences dictate the social preference order.<sup>19</sup> More specifically, to satisfy the nondictatorship condition there can be no person  $j$  such that  $j$ 's individual preference order  $xP_jy$  determines the social preference order  $xPy$  regardless of what other members of society prefer.<sup>20</sup> This condition echoes the democratic intuition that one person's preferences should not dictate policy. If a social choice function violated the condition of nondictatorship, then voting would be pointless: Society could just poll the dictator and implement her preferences.

2. *Pareto efficiency.*

Pareto efficiency stipulates that if everyone prefers alternative  $x$  to alternative  $y$ , then the outcome of the social choice function must also prefer  $x$  to  $y$ .<sup>21</sup> This condition's justification is readily discernible. Democratic elections are intended to settle issues by responding to individual voter preferences. If individual preferences have any meaningful relation to outcomes, then a social choice function that chooses one alternative over another, universally preferred option is perverse. In other words, if every individual agrees that  $x$  is better than  $y$ , a democratic vote should never result in outcome  $y$ . Thus, like nondictatorship, the condition of Pareto efficiency rests upon firm democratic intuitions.<sup>22</sup>

16. ARROW, *supra* note 5, at 51-59. For a more concise version of the proof, see ORDESHOOK, *supra* note 12, at 62-64.

17. See ARROW, *supra* note 5, at 22-31 (establishing his conditions). Riker describes six fairness conditions. RIKER, *supra* note 8, at 116-19. However, his additional criteria—monotonicity and citizens' sovereignty—are variations of the other four. Subsuming these additional conditions into the primary four simplifies the analysis. For a discussion of these additional concepts as independent conditions, see *id.* at 117.

18. See ARROW, *supra* note 5, at 48-51 (proving that the assumption that a single social welfare function could meet Arrow's conditions creates a contradiction). With only two alternatives, a simple majority vote satisfies all of the conditions of fairness and logicity. See *id.* at 48 (observing that this fact "is, in a sense, the logical foundation of the Anglo-American two-party system"). Unfortunately, the real world is never so simple, and society instead relies upon voting devices such as primaries to artificially narrow a voter's range of choices.

19. RIKER, *supra* note 8, at 295.

20. *Id.* at 118.

21. *Id.* at 117.

22. See *id.* at 118 (describing Pareto optimality as "the carrier of monotonicity and nonimposition, both of which have deep and obvious qualities of fairness").

### 3. *Universal admissibility.*

The condition of universal admissibility demands that a social choice function be able to describe a social preference relation for any possible preference profile.<sup>23</sup> Thus, to comply with this condition, a voting procedure must work with all possible permutations of voter preferences over a set of alternatives. For example, given alternatives  $x$ ,  $y$ , and  $z$ , universal admissibility demands that a social choice function operate with the preference profiles of any combination of voters with any of the following preference orders:<sup>24</sup>

- |        |     |     |        |     |     |        |     |     |
|--------|-----|-----|--------|-----|-----|--------|-----|-----|
| 1. $x$ | $y$ | $z$ | 3. $y$ | $x$ | $z$ | 5. $z$ | $x$ | $y$ |
| 2. $x$ | $z$ | $y$ | 4. $y$ | $z$ | $x$ | 6. $z$ | $y$ | $x$ |

The alternative to this condition, restricting individual preference orders, runs counter to democratic principles: People should not be ineligible to vote because of their opinions.<sup>25</sup> Thus, basic notions of democratic fairness demand that social choice procedures operate with any preference profile.

### 4. *Independence from irrelevant alternatives.*

Arrow's fourth fairness condition, independence from irrelevant alternatives, requires that the presence of an irrelevant alternative,  $z$ , in a social preference profile does not affect the order of  $x$  and  $y$  in that profile.<sup>26</sup> The term "irrelevant" is not pejorative; it simply refers to an alternative outside the set from which a group must choose.

The intuition behind this condition is less apparent than with the first three fairness conditions. The following example illustrates the irrationality of allowing irrelevant alternatives to influence preference orders. A waiter offers Joanna a choice between two flavors of frozen yogurt: vanilla and chocolate. Joanna orders vanilla. The waiter takes her order, but quickly returns to inform her that strawberry frozen yogurt is also available.

Joanna responds, "Well, in that case, I'd like chocolate." Joanna's response seems irrational because the existence of strawberry frozen yogurt should not influence her preference for vanilla over chocolate. Strawberry, in other words, is an irrelevant alternative.<sup>27</sup>

Independence from irrelevant alternatives not only ensures rational social preferences, but also prevents manipulation of the social preference order. If

23. *Id.* at 116, 297.

24. As stated in note 10 *supra*, none of the examples in this note considers indifference.

25. RIKER, *supra* note 8, at 117. Riker argues that "[a]ny rule or command that prohibits a person from choosing some preference order is morally unacceptable (or at least unfair) from the point of view of democracy." *Id.*

26. ARROW, *supra* note 5, at 26; RIKER, *supra* note 8, at 118.

27. There are several possible objections to this example. First, Joanna may believe that placing strawberry and vanilla frozen yogurt in the same freezer causes the vanilla to taste awful. But in that case, strawberry is a relevant, not an irrelevant, alternative: Its presence alters the qualitative characteristics of the original choices. Second, Joanna may have changed her mind in the time it took the waiter to return to the table. But this objection merely reflects a flaw in the example. A change in preferences over a period of time does not trigger violations of the condition of independence from irrelevant alternatives on a societal level; only the addition of the irrelevant alternative violates the condition.

the outcome of an election between two alternatives can be altered merely by the introduction or removal of a third alternative, the election is vulnerable to manipulation to achieve a specific result. Thus, the condition of independence from irrelevant alternatives imposes a reasonable requirement upon democratic social choice functions.

### 5. *Transitivity.*

Finally, Arrow's logical condition of transitivity guarantees that a social choice function will produce a complete and transitive social preference order.<sup>28</sup> A transitive arrangement of preferences guarantees that if  $x$  is preferred to  $y$ , and  $y$  to  $z$ , then  $x$  will be preferred to  $z$ .<sup>29</sup> Like independence from irrelevant alternatives, transitivity ensures that social preference orders display some sort of collective rationality.<sup>30</sup> If one prefers beef to chicken, and chicken to fish, it would be inexplicable that he also prefers fish to beef.

Yet perhaps transitive preference orders only serve as a proper condition of rationality for individuals, not groups. For aggregation of individual preference orders that are each transitive may still result in an intransitive social preference order.<sup>31</sup> Although an intransitive individual preference order such as  $xPyPzPx$  signals irrationality, a *social* preference order of the same form may be an acceptable outcome of a social choice procedure.

Intransitive social preference orders, however, suffer from a major problem: their inability to declare a "winner." For example, the social preference order  $xPyPzPx$  fails to designate a clear social choice; each alternative appears to stake an equal claim. In addition, intransitive social orders permit manipulation of social choice through agenda control. Since any alternative in an intransitive social order can prevail if put to a vote at the appropriate moment, control of the voting agenda becomes "tantamount to dictatorial power."<sup>32</sup> For these reasons, the condition of transitivity is essential to ensuring that social choice functions produce meaningful outcomes.

### B. *How Several Popular Social Choice Functions Violate Arrow's Conditions*

If Arrow is correct, no social choice function will satisfy all five conditions of democratic fairness and logicity. This result may constrain legal attempts to structure "fair" voting procedures. The following survey of four social choice procedures illustrates the inevitability of Arrow's conflict.

28. RIKER, *supra* note 8, at 119.

29. *Id.* at 297.

30. *See id.* at 119 (describing the failure to produce social transitivity as "a kind of social irrationality"). According to Riker, Arrow himself described social transitivity as collective rationality. *Id.*

31. *See* FROHLICH & OPPENHEIMER, *supra* note 6, at 27.

32. *Id.* For example, the cycle  $xPyPzPx$  can be manipulated to produce three different outcomes: running  $x$  against  $y$ , with the winner to face  $z$ , results in the social choice  $z$ ; running  $y$  against  $z$ , with the winner to face  $x$ , results in the social choice  $x$ ; and running  $x$  against  $z$ , with the winner to face  $y$ , results in the social choice  $y$ .

### 1. *The Condorcet method.*

The Condorcet method is an adaptation of the more familiar majority decision procedure that allows it to be applied to more than two alternatives.<sup>33</sup> The method places each alternative through a series of simple majority, binary elections with each of the other alternatives.<sup>34</sup> The alternative that defeats each of the other alternatives in every binary comparison is the Condorcet winner.<sup>35</sup> The Condorcet method, therefore, guarantees that if voters prefer one alternative to each of the other alternatives by a simple majority then that alternative becomes the social choice.

If no alternative can defeat each of the others, however, the Condorcet winner remains undefined.<sup>36</sup> Consider the following profile of three voters ranking three alternatives:<sup>37</sup>

	P <sub>1</sub>		
V <sub>1</sub>	x	y	z
V <sub>2</sub>	y	z	x
V <sub>3</sub>	z	x	y

Given this preference profile, the Condorcet method produces the following result:

Votes for the option in the  
row when in contest with  
the option in the column:

	x	y	z
x	-	2	1
y	1	-	2
z	2	1	-

With three voters, an alternative would have to receive at least two votes to defeat another alternative; an alternative must receive at least two votes against all other alternatives to be declared the Condorcet winner. In this example, no single alternative defeats each of the others in a simple majority, binary election. Instead, the Condorcet method produces the voting cycle  $xPyPzPx$ , and the winner remains undefined.<sup>38</sup>

Profile P<sub>1</sub> illustrates Arrow's theorem by demonstrating the Condorcet method's inability to generate a social preference order without violating one of Arrow's five conditions. Preference profiles such as P<sub>1</sub> produce intransitive outcomes, violating Arrow's fifth condition. Individual preference orders that give rise to preference profiles like P<sub>1</sub> must be prohibited to ensure a transitive social preference order. Such a prohibition, however, clearly violates the

33. RIKER, *supra* note 8, at 67.

34. *Id.*

35. *Id.*

36. *Id.* at 67-69.

37. This example is based on Riker's explanation of the Condorcet method and the paradox of voting. *Id.* at 68 display 4-1.

38. *Id.*

condition of universal admissibility. Thus, for the Condorcet method to work, either universal admissibility or transitivity must be abandoned.

## 2. *The amendment procedure.*

The amendment procedure, sometimes called parliamentary voting, is a Condorcet extension<sup>39</sup> designed to select the Condorcet winner, or, if the Condorcet winner is undefined as in  $P_1$ , select the status quo. The amendment procedure presents voters with several alternatives, typically in the form of motions, amendments, or amendments to amendments, in a series of simple majority, binary elections. The winner of the first election competes against the next alternative in a specified order until only one remains. That final alternative then competes against the status quo in a simple majority election.<sup>40</sup>

The amendment procedure, however, may violate the condition of Pareto efficiency. Given some preference profiles, the procedure may select a winner that voters unanimously view as inferior to another alternative.<sup>41</sup> Consider the following profile, composed of the preference orders of three voters over five alternatives:

	$P_2$				
$V_1$	w	x	z	y	s
$V_2$	y	w	x	z	s
$V_3$	s	x	z	y	w

Votes for the option in the row when in  
contest with the option in the column:

	x	y	z	w	s
x	-	2	3	1	2
y	1	-	1	2	2
z	0	2	-	1	2
w	2	1	2	-	2
s	1	1	1	1	-

The only binary election that produces a unanimous winner pairs  $x$  against  $z$ , with  $x$  receiving all three votes. Nonetheless,  $z$  may emerge victorious under the amendment procedure. Consider the following example:

- Step 1:  $x$  vs.  $w$ ;  $w$  wins
- Step 2:  $w$  vs.  $y$ ;  $y$  wins
- Step 3:  $y$  vs.  $z$ ;  $z$  wins
- Step 4:  $z$  vs.  $s$ ;  $z$  wins

Alternative  $z$  emerges as the social choice even though every voter prefers  $x$  to  $z$ . This result clearly violates the condition of Pareto efficiency. To preserve Pareto efficiency would require prohibiting individual preference orders that

39. The following description of the amendment procedure derives its definitions and examples from Riker's work. *Id.* at 69-73.

40. *Id.* at 70.

41. *Id.* at 71-73.



dictate preference profiles such as  $P_2$ . But such a prohibition would, once again, violate the condition of universal admissibility.

### 3. *The Borda count.*

The Borda count is a positional social choice function. Instead of merely evaluating binary relations between alternatives, positional methods consider the ranking of each alternative in an individual preference order. Thus, positional methods such as the Borda count simultaneously consider the ordinal relationships among all of the alternatives, whereas majoritarian methods focus on the ability of one alternative to prevail over another in a binary contest.<sup>42</sup>

The Borda count assigns a numerical score to every alternative in each voter's preference order. In an election with  $n$  alternatives, each voter gives  $n-1$  points to her first choice,  $n-2$  points to her second choice, and continues this process through her last choice, which receives  $n-n$ , or zero, points. Each alternative's scores are summed, and the alternative with the most points becomes the Borda winner.<sup>43</sup>

Given certain preference profiles, however, the Borda count violates the condition of independence from irrelevant alternatives. Consider profile  $P_3$ , reflecting the preference orders of two voters over three alternatives:

	$P_3$		
$V_1$	x	y	z
$V_2$	z	x	y

The Borda count applies to profile  $P_3$  as follows:

	x	y	z
$V_1$	2	1	0
$V_2$	1	0	2
	3	1	2

Since  $x$  receives the most points, it is the Borda winner. The social preference order is  $xPzPy$ .

The Borda count, however, leaves  $P_3$  vulnerable to manipulation by the introduction or removal of irrelevant alternatives.<sup>44</sup> Given preference profile  $P_3$ , the Borda count ranked  $x$  ahead of  $z$ . Yet moving  $y$ 's position within the preference profile can cause  $x$  and  $z$  to reverse rankings in the outcome despite the fact that they maintain the same positions relative to each other. Profile  $P_4$  reflects such a change in the position of alternative  $y$ :

42. *Id.* at 81.

43. *Id.* at 81-82.

44. For an explanation of the irrelevant alternatives criterion, see text accompanying notes 26-27 *supra*. The following discussion draws on Riker's discussion of the Borda count's violation of the independence from irrelevant alternatives criterion. RIKER, *supra* note 8, at 105, 108 display 4-19.

	P <sub>4</sub>		
V <sub>1</sub>	x	z	y
V <sub>2</sub>	z	y	x

The results of applying the Borda count to profile P<sub>4</sub> are:

	x	y	z
V <sub>1</sub>	2	0	1
V <sub>2</sub>	0	1	2
	2	1	3

Every voter has  $x$  and  $z$  in the same order in profiles P<sub>3</sub> and P<sub>4</sub>; the only difference is the positional change of irrelevant alternative  $y$ . But the outcome has changed: Given profile P<sub>4</sub>, the Borda count selects  $z$  instead of  $x$ , producing the social preference order  $zPxPy$ . By selecting  $x$  when given P<sub>3</sub> and  $z$  when given P<sub>4</sub>, the Borda procedure violates the condition of independence from irrelevant alternatives by allowing  $y$  to influence the outcome between  $x$  and  $z$ . Like the Condorcet method and the amendment procedure, the Borda count can only ensure independence from irrelevant alternatives by prohibiting certain individual preference orders, thereby violating the condition of universal admissibility.

#### 4. Cumulative voting.

Cumulative voting, like the Borda count, is a positional social choice procedure.<sup>45</sup> In a cumulative voting scheme, each person is allotted as many votes as there are open seats.<sup>46</sup> Voters may distribute their votes as they see fit, either aggregating their votes for one strongly preferred alternative or dispersing their votes among several alternatives.<sup>47</sup> The alternatives receiving the most votes win.<sup>48</sup>

Cumulative voting, like most positional social choice functions, violates the condition of independence from irrelevant alternatives.<sup>49</sup> Consider the following preference profile, P<sub>5</sub>, in which each of four voters distributes two votes among three alternatives:

45. Lani Guinier, *The Representation of Minority Interests: The Question of Single-Member Districts*, 14 CARDOZO L. REV. 1135, 1156 (1993) (arguing that cumulative voting and other "semiproportional election systems may provide a more politically fair route to participation and political representation for racially distinct groups"). Guinier, a leading proponent of cumulative voting, advocates a proportional power approach to elections within multimember districts as a possible remedy for vote dilution cases and as a tool to revitalize electoral politics. *Id.* at 1169-70.

46. *Id.* at 1169.

47. *Id.* at 1136.

48. Under cumulative voting, a minority comprising 30% of the population could not be prevented from electing a representative of its choice to one of three open seats, provided that members of the minority group voted as a politically cohesive bloc and aggregated their votes. *Id.*

49. See text accompanying notes 26-27 *supra*.

$$P_5$$

(vote distributions in parentheses)

$V_1$	$x(2)$	$y(0)$	$z(0)$
$V_2$	$y(2)$	$z(0)$	$x(0)$
$V_3$	$y(1)$	$z(1)$	$x(0)$
$V_4$	$x(1)$	$z(1)$	$y(0)$

Cumulative voting produces the following outcome:

	$x$	$y$	$z$
$V_1$	2	0	0
$V_2$	0	2	0
$V_3$	0	1	1
$V_4$	1	0	1
	3	3	2

Since  $x$  and  $y$  receive the most votes, they are the cumulative vote winners.

Like the Borda count, however, the addition of irrelevant alternatives subjects cumulative voting to manipulation. The introduction of irrelevant alternative  $w$ , for example, dramatically changes the results. Consider the following profile.

$$P_6$$

(vote distributions in parentheses)

$V_1$	$w(2)$	$x(0)$	$y(0)$	$z(0)$
$V_2$	$w(2)$	$y(0)$	$z(0)$	$x(0)$
$V_3$	$y(1)$	$z(1)$	$x(0)$	$w(0)$
$V_4$	$x(1)$	$z(1)$	$y(0)$	$w(0)$

Cumulative voting produces the following results:

	$x$	$y$	$z$	$w$
$V_1$	0	0	0	2
$V_2$	0	0	0	2
$V_3$	0	1	1	0
$V_4$	1	0	1	0
	1	1	2	4

With the mere addition of irrelevant alternative  $w$ ,  $z$  now beats both of the previous winners, violating the condition of independence from irrelevant alternatives. Once again, the only way to ensure that condition is met is to prohibit certain individual preference orders, violating the condition of universal admissibility.

## II. GENERAL IMPLICATIONS OF ARROW'S THEOREM

### A. Theoretical Implications

Arrow's theorem has profound implications for democratic theory. As the previous analyses of the Condorcet method, amendment procedure, Borda

count, and cumulative voting suggest, no social choice function generates a result consistent with all of Arrow's five conditions. So long as society preserves democratic institutions embodying the four fairness conditions, those institutions will produce intransitive social preference orders.<sup>50</sup> As a result, some social choices will be unordered and thus meaningless.<sup>51</sup> Given that fact, references to "the will of the people" or "the public interest" become suspect because intransitive social preference orders cannot consistently define coherent collective preferences.<sup>52</sup> On initial investigation, then, Arrow's theorem casts doubt upon the usefulness of any social choice procedure and makes the future of democratic theory look bleak indeed.<sup>53</sup>

Proponents of democracy's integrity may raise objections to this dismal forecast. First, perhaps Arrow was wrong. This is unlikely: He sets up only minimal conditions of fairness and logicity, and the proof itself appears invulnerable.<sup>54</sup> Second, even if Arrow's theorem is formally correct, perhaps theorists overstate its negative implications for the future of democratic theory.<sup>55</sup> After all, Arrow's theorem merely proves that no social choice function produces a rational social preference order for *every* preference profile. If certain social choice functions lead to rational outcomes most, or even some, of the time, then there may be less cause for alarm.

Unfortunately, it is impossible to determine the rationality or irrationality of a given social preference order. No secret method of amalgamating individual preferences enables society to determine the "true" social choice. Any standard for evaluating social choices remains vulnerable to the same violations of Arrow's five conditions it is designed to test. Thus, although some social preference orders are both fair and rational, society is incapable of confirming their validity.

Although we cannot verify the reasonableness of any given social outcome, our fear of deceptive outcomes may be minimized by understanding that intransitivities occur infrequently. The next Part addresses this possibility.

50. RIKER, *supra* note 8, at 136.

51. See *id.* at 119, 136. "This conclusion appears to be devastating, for it consigns democratic outcomes—and hence the democratic method—to the world of arbitrary nonsense, at least some of the time." *Id.* at 119.

52. See ORDESHOOK, *supra* note 12, at 56-57 (discussing the ramifications of Arrow's impossibility result in light of the Condorcet paradox).

53. Democratic voting procedures may serve other objectives, such as enhancing governmental legitimacy. However, such justifications ultimately depend on a rational connection between inputs and outcomes: Once people realize that there is no such connection, the other objectives are lost.

54. For a brief proof of the theorem, see FROHLICH & OPPENHEIMER, *supra* note 6, at 23-27. Riker points out that since "the fairness conditions seem intuitively reasonable—at least to people in Western culture—... most of the attack has been focused on logicity." RIKER, *supra* note 8, at 129. He argues that Arrow's theorem nonetheless withstands a critique on the basis of the fairness conditions as well as transitivity. *Id.* at 129-36.

55. Cf. RIKER, *supra* note 8, at 129 (asking whether the theorem either demands too much or overstates the case by stressing the possibility of intransitivity).

## B. *Practical Implications*

The theoretical difficulties described by Arrow's theorem are unavoidable. As illustrated above, democratic decisionmaking procedures inevitably force a choice between universal admissibility and one of the other conditions of fairness or logicity.<sup>56</sup> On a more practical level, however, faith in democratic choice procedures may not be wholly misplaced. The difficulties attendant to Arrow's theorem disappear if preference profiles leading to intransitive social preference orders never occur in the real world. The extent of the practical impact of Arrow's theorem, then, depends upon how often preference profiles prone to cycling actually occur.<sup>57</sup>

Statistically, a substantial proportion of preference profiles result in cycles. In the set of all possible profiles given three voters and three alternatives, 5.6 percent produce voting cycles.<sup>58</sup> As the numbers of voters and alternatives increase, the incidence of cycling approaches 100 percent.<sup>59</sup> It would seem, therefore, that Arrow's theorem actually describes a significant problem in the search for meaningful democratic outcomes.

In reality, however, factors beyond the number of voters and alternatives may help minimize the frequency of cycling. If all voters arrange their alternatives along a common spectrum, cycling will not occur, and a transitive outcome is guaranteed.<sup>60</sup> Consider, for example, an election with three candidates: a conservative (*c*), a moderate (*m*), and a liberal (*l*). Although voters may not support the same candidate, they may very well arrange the candidates along the same political spectrum: *c* on the right side, *l* on the left side, and *m* in the center. This spectrum agreement would imply that the preference profile is "value restricted": All voters agree that one candidate, *m*, is not the worst.<sup>61</sup> Conservative voters would have a preference order of *cPmPl*, liberal voters *lPmPc*, and moderates either *mPlPc* or *mPcPl*. In no case is candidate *m* the least preferred alternative. Thus with complete spectrum agreement, no cycling occurs.

Political and sociological conditions suggest that some degree of spectrum agreement exists in most societies. First, all democracies require a degree of consensus as a precondition to their formation: Absent some agreement, no social contract would exist.<sup>62</sup> Second, common socialization may shape indi-

56. See texts accompanying notes 38, 41, 44 & 49 *supra*.

57. The concept of cycling is closely linked to the logical condition of transitivity. Where a social preference order does not meet the condition of transitivity—in my example, where  $xPyPzPx$ —that order is a "cycle." See RIKER, *supra* note 8, at 294; see also text accompanying notes 28-32 *supra*.

58. RIKER, *supra* note 8, at 122 display 5-1.

59. *Id.* The numbers increase quite rapidly. For example, with five voters and five alternatives, 20% of the possible preference profiles result in voting cycles. *Id.*

60. *Id.* at 123-28. Agreement on the spectrum of alternatives should not be confused with agreement on which alternative is most preferred. The seminal works on the subject of spectrum agreement are DUNCAN BLACK, *THE THEORY OF COMMITTEES AND ELECTIONS* (1963), and DUNCAN BLACK & R.A. NEWING, *COMMITTEE DECISIONS WITH COMPLEMENTARY VALUATION* (1951). For a more concise discussion, see ORDESHOOK, *supra* note 12, at 160-65.

61. RIKER, *supra* note 8, at 128. For the purposes of this example, I assume that a conservative necessarily prefers a moderate to a liberal and that a liberal prefers a moderate to a conservative.

62. FROHLICH & OPPENHEIMER, *supra* note 6, at 19-20.

vidual perceptions of the spectrum of alternatives, producing the type of value restriction that prevents cycles.<sup>63</sup>

Unfortunately, complete spectrum agreement is never guaranteed. Voters may choose to support either extreme over a more centrist position. For example, voters disappointed in a centrist government may prefer both conservative and liberal platforms over moderate proposals. Financially strapped farmers who support substantial farm subsidies may prefer no subsidy over a 30 percent proposal since a complete lack of support would at least facilitate their decision to switch occupations.

The likelihood of achieving spectrum agreement decreases when candidates, not discrete issues, comprise the array of alternatives. Since candidates take positions on many different issues, the participation of single-issue voters will make spectrum agreement unlikely, as voters exhibit different profiles according to their particular issue preference. In practice, then, social choice procedures will encounter preference profiles containing groups of voters who disagree on the spectrum of alternatives. Without spectrum agreement, the validity of social preference orders remains uncertain.

The practical significance of Arrow's theorem, then, is twofold. First, the enormous theoretical import of the theorem affects the real world of democratic social choice procedures to the extent that voters fail to agree on the spectrum of alternatives. This implies that the efficacy of any social choice function hinges upon the existence of spectrum agreement. Second, the fact that social preference functions cannot eliminate the possibility of intransitive social preference orders requires close monitoring of the agenda setting process. Any alternative within a voting cycle can become the social choice if presented at an opportune time;<sup>64</sup> thus, the individual or group that controls the agenda can effectively dictate the social choice.

### C. *Implications for Voting Rights*

Beyond its powerful theoretical and practical implications, what lessons does Arrow's theorem offer students of voting rights? The remainder of this note explores that question and suggests ways in which the theorem might contribute to the search for judicially manageable standards for measuring vote dilution.

#### 1. *Vote dilution and Arrow's theorem.*

Courts first started to struggle with vote dilution after the passage of the Voting Rights Act of 1965.<sup>65</sup> Even after the battle to increase minority access to the voting booth was largely won, devices such as racial gerrymandering and at-large elections continued to limit minority representation by effectively dilut-

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63. *See id.* at 20.

64. *See* note 32 *supra* and accompanying text.

65. 42 U.S.C. §§ 1971, 1973-1973p (1988). For a discussion of the 1965 Act's content, later amendments, and impact on black political participation, see BERNARD GROFMAN, LISA HANDLEY & RICHARD G. NIEMI, *MINORITY REPRESENTATION AND THE QUEST FOR VOTING EQUALITY* 15-23 (1992).

ing minorities' voting power.<sup>66</sup> Therefore, voting rights advocates turned to these more invidious forms of discrimination.<sup>67</sup>

Although the issue of vote dilution pervades voting rights litigation, neither courts nor commentators have yet articulated an accepted definition.<sup>68</sup> A helpful definition would set up a standard against which to measure dilution. In order to ascertain when minorities "have less opportunity than other members of the electorate to participate in the political process and to elect representatives of their choice,"<sup>69</sup> one must first determine what the outcome should look like in the absence of dilution. Thus, the search for a definition of vote dilution is equivalent to the search for "the ideal against which vote dilution is identified and measured."<sup>70</sup>

For quantitative vote dilution, dividing the total population by the total number of representatives establishes a district's standard. Comparing the size of an actual district to the district standard reveals the extent of quantitative dilution. If a significant deviation comes to light, certain adjustments can remedy the situation.<sup>71</sup> This relatively simple process for identifying quantitative vote dilution contrasts sharply with the more intractable problem of finding and solving qualitative vote dilution.

Commentators and courts have proposed various standards for measuring qualitative vote dilution. In the context of minority vote dilution, for example, one suggested alternative would establish a standard of proportional representation wherein minorities would constitute the same proportion of members in a legislative body as they do in the general population.<sup>72</sup>

Unfortunately, the difficulties Arrow's theorem spells out for social choice functions also accompany attempts to develop a standard for evaluating those functions. Deriving a standard requires either an implicit or explicit equating of preference profiles with ideally matched social preference orders.<sup>73</sup> Yet the social choice function selected for matching preferences with a social preference order remains vulnerable to the theoretical hazards of Arrow's theorem. Thus, without a method of finding the "correct" social outcome, commentators

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66. See Grofman et al., *supra* note 65, at 24 ("[A]lthough blacks might vote, they would often be unable to elect candidates of their choice.").

67. See, e.g., *id.* at 23-24 (enumerating more subtle schemes for reducing minority voter participation, such as at-large elections, anti-single-shot laws, decreases in the size of legislative bodies, racial gerrymandering, and exclusive slating).

68. The watershed case for vote dilution challenges is *Thornburg v. Gingles*, 478 U.S. 30 (1986). Writing for the majority, Justice Brennan affirmed that a "totality of the circumstances" test applies to vote dilution and set up three necessary preconditions for such a finding under § 2 of the Voting Rights Act. *Id.* at 50-51, 79. However, he failed to define vote dilution.

69. 42 U.S.C. § 1973(b) (1988).

70. Alexander, *supra* note 1, at 567.

71. See, e.g., *Reynolds v. Sims*, 377 U.S. 533, 568 (1964) (requiring the apportionment of state electoral districts by population).

72. Under the Voting Rights Act, evidence of disproportionate representation, while not dispositive, may help substantiate a vote dilution claim. 42 U.S.C. § 1973(b) (1988). However, the Act explicitly states that no one enjoys a right to proportional representation. *Id.*

73. A proportional representation standard satisfies this condition since it implies racial bloc voting or some other common preference grouping. Proportional representation is pointless if voters are indifferent to being represented by minority representatives.

and courts cannot even measure qualitative vote dilution, much less remedy it. Accordingly, attempts to develop standards for measuring vote dilution are theoretically doomed from the outset.

Several characteristics of complex representative democracies further complicate the search for qualitative vote dilution standards. First, a voter's preferences may vary according to the district in which she is placed.<sup>74</sup> Even in the presence of reliable polling, such fluidity in preferences, particularly when coupled with the possibility of strategic voting,<sup>75</sup> obfuscates individual preference orders.

Second, and perhaps more important, the characteristics of modern representative democracies increase the likelihood of intransitive social preference orders: The morass of conflicting issue and candidate preferences frustrates attempts to filter out a principled standard of qualitative vote dilution. As one commentator notes:

As voters we are Democrats and Republicans, blacks and whites, males and females. But we are also hawks and doves, redistributionists and laissez-faire advocates. We are atheist, agnostic, Catholic, Protestant, Jewish, Muslim, and Buddhist, all of various stripes. We are trade unionists and managers, Main Streeters and cosmopolites. Some of us prefer hot, charismatic candidates; others prefer cooler types. Some of us prefer the well-educated or the well-bred. Others prefer regular Joes and Joans. The list of our voting-relevant divisions is virtually endless.<sup>76</sup>

This multiplicity of personal preferences and social influences makes a natural occurrence of spectrum agreement unlikely. Voters can and do cross race and party lines for a variety of reasons. The resulting lack of spectrum agreement increases the likelihood of intransitive outcomes. Thus, an effective solution to the problem of qualitative vote dilution requires a standard that accommodates individuals who align with different candidates for divergent reasons. However, the dictates of Arrow's theorem throw a serious roadblock in front of any such effort.

## 2. *Reactions to the problems raised by Arrow's theorem.*

Larry Alexander contends that in a modern representative democracy where voters often cross race and party lines, Arrow's theorem implies that there is no method to measure vote dilution beyond the quantitative standard of "one person, one vote."<sup>77</sup> Either no person can prove qualitative vote dilution or, if one can, every person can demonstrate dilution by characterizing his or her vote as

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74. See Alexander, *supra* note 1, at 572-73 (illustrating how a voter's preferences may differ depending whether she is in a district that is ethnically homogeneous or heterogeneous).

75. An individual voting strategically votes against her true preferences, intending to bring about a social choice she desires more than the one that could otherwise be reached if she voted in accordance with her true preferences. For example, in states with open primaries, a Democrat could vote for her least preferred candidate in the Republican primary to help ensure the ultimate success of the Democratic candidate in the general election. For a good theoretical discussion of strategic voting, see ORDESHOOK, *supra* note 12, at 82-89. For historical examples, see RIKER, *supra* note 8, at 141-56.

76. Alexander, *supra* note 1, at 575.

77. *Id.* at 575-76.



part of some conceivable group interest.<sup>78</sup> Without a neutral procedural method of identifying qualitative vote dilution, any remedy courts devise will actually be a substantive political decision.<sup>79</sup> In a sense, courts will act as dictators, usurping legislative authority and setting up agendas and districts to achieve their preferred social preference orders. Since courts cannot determine when qualitative vote dilution has occurred, Alexander advocates that they do "absolutely nothing."<sup>80</sup>

However, Alexander's declaration of the death of qualitative vote dilution may be premature. Arrow's theorem does not wholly invalidate attempts to derive standards for qualitative vote dilution. Rather, the theorem proposes that no social choice function, and hence no qualitative vote dilution standard, can satisfy *all* of Arrow's conditions of fairness and logicity. Thus instead of completely abandoning the search for a qualitative vote dilution standard, courts should follow a more pragmatic course and recharacterize the search in terms of the theorem. More specifically, when devising a qualitative vote dilution standard, courts should decide which of Arrow's five conditions to surrender.

### III. PRAGMATIC ACCOMMODATIONS WITH ARROW'S THEOREM

#### A. *Lessons for Voting Rights Advocates*

##### 1. *Devise and apply standards in limited circumstances.*

Courts and legislatures should devise and apply standards for qualitative vote dilution only in limited circumstances. More specifically, a standard should be applied only to a limited number of groups within districts that display spectrum agreement.

Limiting application of a qualitative vote dilution standard to a few groups prevents some of the practical problems attendant to Arrow's theorem. Indeed, to avoid the potential complications the theorem describes, legislatures and courts should be wary of extending standards to more than one group within a district at a time. Voting cycles are more often found in districts with a wide array of individual preference orders and complex voting patterns.<sup>81</sup> Individuals in complex representative democracies have as many claims for vote dilution as there are groups of which they are members. Focusing upon any particular group will inevitably dilute the vote of another; focusing upon all groups is theoretically impossible. Since a districting plan that benefits one group most likely dilutes others' voting power,<sup>82</sup> attempts to devise a standard applicable to a large number of groups are destined to fail. Legislatures and

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78. *Id.* at 576. Alexander raises a related issue: "If we cannot determine whom a voting scheme actually hurts, who should have standing to challenge it?" *Id.* at 577.

79. *Id.* at 578-79.

80. *Id.* at 577-78. Alexander would instead leave the choice of how to district to "the imperfect but more accountable political mechanism, the legislature." *Id.* at 578.

81. See notes 58-59 *supra* and accompanying text.

82. Alexander, *supra* note 1, at 576-77.

courts should therefore limit the number of groups that may make vote dilution claims in any particular district.

Having limited the number of groups that may claim vote dilution, the courts' inquiries should shift to the more substantive issue of prioritizing claims of vote dilution. In keeping with the post-Civil War amendments<sup>83</sup> and the Voting Rights Act of 1965,<sup>84</sup> claims by the historically disadvantaged minorities they target should probably receive first priority. Secondary groups' claims should receive consideration only if a district lacks a significant number of historically disadvantaged minorities. In any case, prioritization should strictly limit the number of possible dilution claims. As suggested above, a large number of groups can quickly overwhelm the search for qualitative vote dilution standards.

Larry Alexander, the advocate of judicial inaction in the vote dilution arena,<sup>85</sup> considers the option of limiting dilution claims to a few historically disadvantaged groups. However, he concludes that historically disadvantaged minorities do not form a cohesive voting bloc and that, consequently, a vote dilution standard for even a small number of groups will be prone to intransitivities.<sup>86</sup> This response is telling, since Alexander does not completely reject the notion of devising a vote dilution standard. Instead, he merely argues that focusing on one minority group is not a sufficient condition for devising a workable standard: A meaningful vote dilution standard must apply where targeted minority groups also form cohesive voting blocs. Alexander's admission directs us to the second limiting principle implied by Arrow's theorem: the need for spectrum agreement.<sup>87</sup>

Legislatures and courts should primarily focus on the qualitative vote dilution claims within districts demonstrating some degree of spectrum agreement. Such agreement decreases the risk of intransitive outcomes, permitting courts to develop qualitative vote dilution standards that do not violate Arrow's basic conditions of fairness and logicity.<sup>88</sup> Without spectrum agreement, the risk of intransitivities makes any qualitative vote dilution standard relatively meaningless. In short, although spectrum agreement in a multifaceted, representative democracy is rare, the risk of intransitive outcomes declines to the extent that spectrum agreement does exist. Therefore, qualitative vote dilution standards, and the legal cases built around them, are strongest where there is some agreement on the spectrum of alternatives.

Reliance upon spectrum agreement to generate a transitive vote dilution standard, however, sacrifices Arrow's condition of universal admissibility. By definition, spectrum agreement implies the absence of certain individual preference orders, and thus involves a clear violation of universal admissibility. Even

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83. U.S. CONST. amends. XIII-XV.

84. 42 U.S.C. §§ 1971, 1973-1973p (1988).

85. See notes 77-80 *supra* and accompanying text.

86. Alexander, *supra* note 1, at 579.

87. See notes 60-64 *supra* and accompanying text.

88. See RIKER, *supra* note 8, at 128 ("If . . . voters have a common view of the political [spectrum] . . . then a transitive outcome is guaranteed.").

a social choice procedure operating on a profile with naturally occurring spectrum agreement may violate universal admissibility, since that condition requires that a social choice function generate a complete transitive outcome for any *possible* preference profile.

As discussed in Part I, compromising the condition of universal admissibility jeopardizes a basic element of democratic fairness.<sup>89</sup> Natural spectrum agreement, however, satisfies the fairness concerns embodied by that condition. In cases of natural spectrum agreement, voters face no prior restraints on their preference orders. As a result, natural spectrum agreement does not implicate the principal justification for universal admissibility, the immorality of denying the ballot to people with certain preference orders. Cases of natural spectrum agreement instead indicate that voters' individual preferences happen to align along a common spectrum. Thus, universal admissibility is not sacrificed by denying anyone the right to vote from the outset, but by determining when enough spectrum agreement exists to make cycling unlikely. In short, to sacrifice universal admissibility in a value-restricted district is to sacrifice very little.

On a practical level, courts should focus their efforts on districts that exhibit some form of bloc voting. Bloc voting makes spectrum agreement more likely: Both majority and minority voters may have preferences that align along some issue spectrum.<sup>90</sup> For example, in a racially polarized voting district with white (*w*) and black (*b*) candidates, voters have preferences like *bPbPw*, *bPwPw*, *wPwPb*, and *wPbPb*, rather than *bPwPb* or *wPbPw*.<sup>91</sup> The candidate's race determines whether he or she appears at the beginning or the end of individual preference orders. Bloc voting thus offers an example of a natural value restrictedness that decreases the likelihood of intransitivities without imposing prior restraints upon individual preferences. In sum, since vote dilution standards can only avoid the strictures of Arrow's theorem in limited circumstances, courts should target their attempts to devise standards on a small number of groups located within districts exhibiting a great degree of spectrum agreement.

## 2. *Pay attention to agenda control.*

Courts and legislatures must guard against agenda manipulation designed to control outcomes.<sup>92</sup> Intransitive social preference orders permit the agenda setter to act as a dictator.<sup>93</sup> As Norman Frohlich and Joe Oppenheimer point out,

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89. See note 25 *supra* and accompanying text.

90. A more sophisticated analysis of racial bloc voting would examine spectrum agreement regarding a candidate's position on issues affecting race as well as his or her actual race.

91. Racial spectrum agreement should not be confused with agreement upon the candidates. With racial spectrum agreement, although white voters will prefer white candidates, and black voters black candidates, both groups agree that the candidates are arrayed on the same basic spectrum. In other words, whites rest on one side, and blacks on the other.

92. See texts accompanying notes 32 & 64 *supra*. This second lesson of Arrow's theorem carries broad implications. For example, policymakers should carefully supervise both primary and general elections. Indeed, Arrow's theorem suggests that the processes leading up to the final election should be monitored as closely as the final election itself.

93. See note 32 *supra* and accompanying text.

"[t]he possibility of intransitive social choices means that the order in which issues are put to a vote determines which alternative will be adopted."<sup>94</sup> For example, in the case of the cycle  $xPyPzPx$ , any alternative can prevail depending on the particular sequence of binary elections used to determine the winner. In other words, pitting  $x$  against  $y$ , with the winner to face  $z$ , results in the social choice  $z$ ; pitting  $y$  against  $z$ , with the winner to face  $x$ , results in the social choice  $x$ ; and pitting  $x$  against  $z$ , with the winner to face  $y$ , results in the social choice  $y$ . Manipulation of the agenda is tantamount to manipulation of the outcome.

Transitive social preference orders provide the best protection against agenda manipulation. Yet while securing spectrum agreement is one way to assure such transitivity, even the most careful attention to the presence of such agreement may not be enough. Indeed, the difficulty of confirming and maintaining spectrum agreement may thwart efforts to prevent agenda manipulation. First, practical difficulties plague the process of measuring spectrum agreement.<sup>95</sup> Second, voter preferences may unexpectedly shift and realign under new issues. Finally, spectrum agreement based on minority status may dissipate with advances in social justice. These difficulties are all exacerbated in important elections, since the incentive to manipulate becomes greater when more is at stake. Such prospects make intransitive results possible and even likely, which in turn opens the door to agenda manipulation.

#### B. *Legislative and Judicial Incorporation of the Lessons of Arrow's Theorem*

The principal legislative and judicial formulations of vote dilution standards appear to incorporate the main lessons of Arrow's theorem. Plaintiffs have two avenues to pursue vote dilution claims. First, plaintiffs may claim that a diluting electoral device violates the Fourteenth or Fifteenth Amendment of the Constitution.<sup>96</sup> Second, plaintiffs may seek the remedy provided by sections 2 and 5 of the Voting Rights Act.<sup>97</sup> Rather than survey the entire array of vote dilution remedies, the following Part will focus on section 2 of the Voting Rights Act, as amended in 1982, and its subsequent interpretation by the Supreme Court in *Thornburg v. Gingles*.<sup>98</sup>

94. FROHLICH & OPPENHEIMER, *supra* note 6, at 27.

95. "[R]acially polarized voting is not self-evident. Because individual voting records are secret, one cannot immediately determine whether minorities and whites vote for the same or different candidates." GROFMAN ET AL., *supra* note 65, at 82; *see id.* at 82-108 (detailing many of the problems and methodologies associated with measuring racial bloc voting).

96. *See, e.g., Reynolds v. Sims*, 377 U.S. 533, 568 (1964) (holding that vote dilution violates the Equal Protection Clause of the 14th Amendment).

97. 42 U.S.C. §§ 1973, 1973c (1988). These sections establish a cause of action for those protected classes whose voting rights are abridged by improper electoral maneuvering. Section 5 also provides for prior approval by the Attorney General of legislative apportionment schemes for jurisdictions with a history of electoral discrimination. *Id.* § 1973c.

98. 478 U.S. 30, 79 (1986); *see* note 68 *supra*.

### 1. Section 2 of the Voting Rights Act of 1965.

The 1982 amendments to the Voting Rights Act make vote dilution an actionable offense. On its face, section 2 of the Act<sup>99</sup> seems to incorporate the lessons of Arrow's theorem. For example, it only permits dilution claims based on race or color. In most districts, this requirement significantly limits the number of groups that may make qualitative vote dilution claims. By legitimizing the use of minority representation levels as an index of vote dilution, section 2 also underscores the importance of spectrum agreement in developing effective vote dilution claims.<sup>100</sup> Furthermore, by considering "the political processes leading to nomination or election,"<sup>101</sup> section 2 acknowledges the power of agenda control. In short, the text of the Voting Rights Act, and of section 2 in particular, generally accords with the dictates of Arrow's theorem.

The extensive legislative history of the 1982 amendments also comports with the lessons of Arrow's theorem. The report by the Senate Committee on the Judiciary declared that proof of a section 2 violation may include several specified indicia of dilution, collectively known as the "Senate factors."<sup>102</sup>

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99. Section 2 reads as follows:

(a) No voting qualification or prerequisite to voting or standard, practice, or procedure shall be imposed or applied by any State or political subdivision in a manner which results in a denial or abridgement of the right of any citizen of the United States to vote on account of race or color, or in contravention of the guarantees set forth in section 1973b(f)(2) of this title, as provided in subsection (b) of this section.

(b) A violation of subsection (a) of this section is established if, based on the totality of circumstances, it is shown that the political processes leading to nomination or election in the State or political subdivision are not equally open to participation by members of a class of citizens protected by subsection (a) of this section in that its members have less opportunity than other members of the electorate to participate in the political process and to elect representatives of their choice. The extent to which members of a protected class have been elected to office in the State or political subdivision is one circumstance which may be considered: *Provided*, That nothing in this section establishes a right to have members of a protected class elected in numbers equal to their proportion in the population.

42 U.S.C. § 1973 (1988).

100. Since complete agreement on a race spectrum usually results in the disproportionate election of majority candidates under most district voting procedures, the statute allows courts to consider relative levels of representation as evidence of vote dilution. *Id.* § 1973(b); see note 72 *supra*.

101. 42 U.S.C. § 1973(b) (1988) (emphasis added).

102. The complete text of the Senate factors appears in VOTING RIGHTS ACT EXTENSION: REPORT OF THE COMMITTEE ON THE JUDICIARY, UNITED STATES SENATE, ON S. 1992 WITH ADDITIONAL, MINORITY, AND SUPPLEMENTAL VIEWS, S. REP. NO. 417, 97th Cong., 2d Sess. 28-29 (1982). Those factors include:

1. the extent of any history of official discrimination in the state or political subdivision that touched the right of the members of the minority group to register, to vote, or otherwise to participate in the democratic process;

2. the extent to which voting in the elections of the state or political subdivision is racially polarized;

3. the extent to which the . . . subdivision has used [any of various] voting practices or procedures that may enhance the opportunity for discrimination against the minority group;

4. if there is a candidate slating process, whether the members of the minority group have been denied access to that process;

5. the extent to which members of the minority group in the . . . subdivision bear the effects of discrimination in [other areas] which hinder their ability to participate effectively in the political process;

6. whether political campaigns have been characterized by overt or subtle racial appeals;

Significantly, several of these factors closely parallel the requirements of Arrow's theorem.

Senate factors two, six, and seven help ensure the presence of spectrum agreement. Factor two, "the extent to which voting in the elections . . . is racially polarized,"<sup>103</sup> provides a fairly direct measure of spectrum agreement. Senate factor six considers "whether political campaigns have been characterized by overt or subtle racial appeals."<sup>104</sup> In a campaign marred by racial appeals, candidates may either believe that a significant amount of racial spectrum agreement exists or that such agreement can be fostered. Factor seven, "the extent to which members of the minority group have been elected to public office in the jurisdiction,"<sup>105</sup> relates to racial bloc voting since, under most district voting procedures, complete agreement on a racial spectrum will result in the election of majority candidates in disproportionate numbers.

Senate factors three and four concentrate on agenda control. Factor three reviews the extent to which a jurisdiction exhibits discrimination-enhancing electoral procedures.<sup>106</sup> Senate factor four examines "whether the members of the minority group have been denied access to [the candidate slating] process."<sup>107</sup> By focusing on attempts to control the agenda, either during the slating process or the election itself, factors three and four direct attention to a particularly dangerous aspect of social choice procedures. Through addressing spectrum agreement and agenda manipulation, the language and legislative history of section 2 reflect some of the basic lessons derived from Arrow's theorem.

## 2. *Thornburg v. Gingles*.

In its first major interpretation of the 1982 amendment to section 2, the Supreme Court in *Thornburg v. Gingles*<sup>108</sup> focused significant attention on racial bloc voting in analyzing qualitative vote dilution. Justice Brennan, writing for the majority in *Thornburg*, devised a three-pronged test to identify vote dilution in multimember districts:

These circumstances are necessary preconditions for a [violation of section 2]. First, the minority group must be . . . sufficiently large and geographically compact to constitute a majority in a single-member district. . . . Second, the minority group must be . . . politically cohesive. . . . Third, the minority must be able to demonstrate that the white majority votes sufficiently as a bloc to enable it . . . usually to defeat the minority's preferred candidate.<sup>109</sup>

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7. the extent to which members of the minority group have been elected to public office in the jurisdiction.

*Id.* (footnotes omitted).

103. *Id.* at 29.

104. *Id.*

105. *Id.*

106. *Id.* Factor three lists "unusually large election districts, majority vote requirements, [and] anti-single shot provisions" as examples of "voting practices or procedures that may enhance the opportunity for discrimination against the minority group." *Id.*

107. *Id.*

108. 478 U.S. 30 (1986); see note 68 *supra*.

109. *Thornburg*, 478 U.S. at 50-51 (citations omitted).

Taken together, Justice Brennan's conditions address several of the major concerns raised by Arrow's theorem.

The first condition's emphasis on size and compactness basically requires a minority interest worthy of protection. This condition effectively limits the number of groups with a legally cognizable claim of vote dilution. Together, the second and third conditions, which measure minority and majority bloc voting, make racially polarized voting a precondition for vote dilution claims. In other words, in *Thornburg* the Supreme Court made spectrum agreement a necessary condition for proving qualitative vote dilution.

Justice Brennan, however, failed to clearly articulate the underlying reason for making racially polarized voting a precondition to dilution claims. Grofman suggests that Justice Brennan may have merely reiterated the rationales expressed by the Fifth and Eleventh Circuits in previous vote dilution cases:<sup>110</sup> "Unless there is an initial showing of significant racial bloc voting, the other factors will not demonstrate that the plaintiffs have suffered a substantial inability to elect their preferred candidates."<sup>111</sup> This statement, however, begs the question *why* courts should demand a showing of racial bloc voting in vote dilution claims.

Arrow's theorem provides one possible answer to the question Justice Brennan left open. Namely, showing qualitative vote dilution requires comparing an actual outcome against a standard, an ideal outcome. Identifying the standard depends upon confirming the existence of transitive social preference orders. Yet Arrow's theorem shows that attempts to derive a standard will violate at least one fairness condition. The presence of a natural spectrum agreement, however, guarantees the existence of a transitive social preference order and thus helps identify the ideal outcome without sacrificing the underlying fairness concerns of universal admissibility.<sup>112</sup> Since racial bloc voting is a form of natural spectrum agreement, the presence of racially polarized voting may justifiably be deemed a critical precondition to qualitative vote dilution claims.

### CONCLUSION

Justice Frankfurter did not overstate his warning to courts regarding the dangers inherent in the "political thicket" of legislative apportionment. Indeed, recent judicial forays into that thicket have increasingly become entangled in an imposing theoretical dilemma. The dilemma Arrow's theorem describes may be a fundamental cause of the Supreme Court's inability to devise an "ideal"

110. GROFMAN ET AL., *supra* note 65, at 50. The 5th Circuit previously emphasized the importance of racial polarization in vote dilution cases. *See, e.g., Jones v. City of Lubbock*, 727 F.2d 364, 379-80 (5th Cir. 1984) (affirming that the test for vote dilution relies on election results). The 11th Circuit in *United States v. Marengo County Comm'n*, 731 F.2d 1546 (11th Cir.), *cert. denied and appeal dismissed*, 469 U.S. 976 (1984), emphasized the significance of racial polarization to a § 2 vote dilution standard: "[T]his factor will ordinarily be the keystone of a dilution case." *Id.* at 1566; *cf. United States v. Dallas County Comm'n*, 850 F.2d 1433, 1439-40 (11th Cir. 1988) (factoring the history of racially polarized voting in Dallas County into the court's analysis of a proposed remedial voting plan), *cert. denied*, 490 U.S. 1030 (1989).

111. GROFMAN ET AL., *supra* note 65, at 50.

112. *See* notes 23-25 & 60-64 *supra* and accompanying texts.

outcome against which to measure vote dilution. An ideal standard should satisfy some basic conditions of fairness and logicity—nondictatorship, Pareto efficiency, universal admissibility, independence from irrelevant alternatives, and transitivity. Arrow's theorem, however, dictates that no standard can satisfy all of those conditions. Rather, the only way to guarantee meaningful, transitive outcomes is to sacrifice at least one fairness condition.

Although Arrow's theorem appears to conceptually preclude the existence of fair and logical vote dilution standards, recharacterizing the search in terms of the theorem may provide a solution. Courts should focus on those circumstances in which the sacrifice of one of Arrow's conditions is the least distasteful. Spectrum agreement guarantees a transitive outcome but sacrifices universal admissibility. Natural spectrum agreement in the form of racial bloc voting, however, enables courts to devise meaningful vote dilution standards without imposing prior restraints on individuals' preferences. This approach sacrifices universal admissibility without unduly burdening democratic fairness.

By mapping the landscape of the problem of qualitative vote dilution, Arrow's theorem can guide decisionmakers in devising new standards. Recent decisions by courts and legislatures generally comport with the principles and dictates of the theorem. Arrow's theorem, then, may help the Supreme Court light a path out of a difficult political thicket.



